Dr. Anjali Kaushik
Sr. Lecturer
Accuman Institute of Management,
Greater Noida

Multi-Objective Transportation Planning Problems

Abstract

The basic assumption underlying most of the formulations to transportation model is that the decision-maker is concerned with optimum value of only one objective, i.e. minimum total transportation cost. However, in practice, economic optimization is not the only objective of management inenterprises. In fact, decision-maker quite frequently place higher priorities on other non-economic goals that are vital to the existance of his firm than the objective of cost minimization, and seeks cost minimization while pursuing other non-economic objectives also.

The purpose of this study is to illustrate how linear goal programming can be used as an aid in solving a transportation problem with various other considerations along with cost minimization. The data for the study was collected from an Oil Company.

INTRODUCTION

In business and industry, a variety of methods such as transportation algorithm, linear programming and generalised minimum cost network algorithms have been used for solving transportation problems. The objective of all these methods is to minimise the total transportation cost. The basic assumption underlying most of the formulations of these transportation models is that management is concerned solely with one objective, namely cost minimization. However, in practice, economic optimization is not the only objective of management in enterprises [4]. In fact, management quite frequently place higher priorities on other non-economic goals that are vital to the existence of their firms than the objective of cost minimization, and they seek cost minimization while pursuing other noneconomic objectives. These diverse objectives also apply to the transportation problem.

In the transportation problem, there may be multiple and conflicting objectives such as: provision of a stable employment level in various plants and transportation fleet, balancing work among the plants, cost minimisation, and satisfying union demand, etc. In this study, the linear goal programming approach is used that provides an analytical framework by which a decision-maker can optimise multiple and conflicting objectives.

Kwak and Sehniederajans [2, 3] applied linear goal programming as an aid to resolving a transprotation problem with variable supply and demand requirements. The purpose of this study is to illustrate how linear goal programming can be used as an aid to solving transportation problems with various union considerations. The data used in our transportation problemn is collected from an oil company.

THE MODEL

In order to demonstrate the model, the variables are defined as follows:

$x_{i,j}$ = amount to be transported from the plant'*i* to the plant'*j* to the depot'*j*.

d_i^- = undeachievement of constraints in the *ith* equation.

d_i^+ = overachievement of constraints in the *ith* equation.

THE PROBLEM

The oil company used in this study produces a single oil product at its three plants located in three cities in three states and supplies the product from the three plants to twenty depots located in five states in northern India. To maintain confidentiality, the name of company and depots are not provided in this study. The monthly production capacities of oil product at the three plants are given in Table 1.

Table 1 Monthly Production Capacity of Each Plant

Plant	Capacity (tons)
1	1300
2	1500
3	1600
Total	4400

The policy of the company in the past has been to solve transportation problem by using standard transportation algorithm or by adopting a standard linear programming problem, with the primary goal of transportation cost minimization, and all other goals specified as constraints. However, because companies are in most cases, faced with multiple objectives, an alternative technique of using linear goal programming (LGP) model has been adopted. Even though, all goals may not be exactly achieved under this technique, it provides the closest optimal solution, given the constraints of the problem.

THE GOALS

Various goals set by the management in order of their importance were as follows:

(i) Satisfy 100% demand requirement for depot 18.

(ii) Minimise the amount transported from plant 1 to depot 1 to no more than 100 tons.

(iii) Minimise the total amount transported to each depot no more than 80% of the demand.

(iv) Minimise the total transportation cost to no more than Rs. 5,90,415

(v) Minimise shipment of goods from plant 3 to depot 5.

(vi) Minimise demand deviations between depot 2 and 7.

(vii) Minimise the total transportation costs for goods shipped.

A summary of monthly demand of each depot and cost per ton from each plant is given in Table 2

Table 2 Summary of Data

S.No	*From Plant (cost per ton in Rs)*			*To depot*	*Demand (tons)*
	1	2	3		
1.	170	340	100	1	200
2.	100	270	100	2	100
3.	120	230	160	3	100
4.	110	280	200	4	100
5.	225	160	350	5	200
6.	310	110	400	6	200
7.	170	340	280	7	300
8.	290	160	370	8	200
9.	300	150	400	9	100
10.	310	140	375	10	200
11.	300	130	400	11	100
12.	340	125	410	12	200
13.	295	150	420	13	200
14.	290	160	390	14	100
15.	360	180	450	15	250
16.	340	380	160	16	150
17.	350	450	150	17	150
18.	67	300	250	18	500
19.	250	400	67	19	700
20.	300	67	400	20	700
				Total	**4750**

GOAL CONSTRAINTS

The linear GP model constraints for the transportation probelm are formulated as follows.

(i) The supply is restricted to the miximum capacity of plants. Since it is assumed that the right-hand side values indiate the maximum capacity of the plant, positive deviations can be excluded from the supply constraints. The LGP constraints for supply are given as follows:

$$\sum_{j=1}^{20} X_{1_j} + d_1^- = 1300$$

$$\sum_{j=1}^{20} X_{2_j} + d_2^- = 1500$$

$$\sum_{j=1}^{20} X_{3_j} + d_3^- = 1600$$

(ii) Since the Company never wisher to overfill a depot's demand, positive deviations can be excluded from demand constraints. However, since demand cannot be satisfied in all cases, negative deviations must be included to identify the underachievement of gemand goals. The LGP constrains for demand are given below:

$$\sum_{j=1}^{3} X_{i,1} + d_4^- = 200$$

$$\sum_{j=1}^{3} X_{i,2} + d_5^- = 100$$

$$\sum_{j=1}^{3} X_{i,3} + d_6^- = 100$$

$$\sum_{j=1}^{3} X_{i,4} + d_7^- = 100$$

$$\sum_{j=1}^{3} X_{i,5} + d_8^- = 100$$

$$\sum_{j=1}^{3} X_{i,6} + d_9^- = 200$$

$$\sum_{j=1}^{3} X_{i,7} + d_{10}^- = 300$$

$$\sum_{j=1}^{3} X_{i,8} + d_{11}^- = 200$$

$$\sum_{j=1}^{3} X_{i,9} + d_{12}^- = 100$$

$$\sum_{j=1}^{3} X_{i,10} + d_{13}^- = 200$$

$$\sum_{j=1}^{3} X_{i,11} + d_{14}^- = 100$$

$$\sum_{j=1}^{3} X_{i,12} + d_{15}^- = 200$$

$$\sum_{j=1}^{3} X_{i,13} + d_{16}^- = 200$$

$$\sum_{j=1}^{3} X_{i,14} + d_{17}^- = 100$$

$$\sum_{j=1}^{3} X_{i,15} + d_{18}^- = 250$$

$$\sum_{j=1}^{3} X_{i,16} + d_{19}^- = 150$$

$$\sum_{j=1}^{3} X_{i,17} + d_{20}^- = 150$$

$$\sum_{j=1}^{3} X_{i,18} + d_{21}^- = 500$$

$$\sum_{j=1}^{3} X_{i,19} + d_{22}^- = 770$$

$$\sum_{j=1}^{3} X_{i,20} + d_{23}^- = 700$$

(iii) Management/union agrement specifies that at least 100 tons be transported from plant 1 to depot 1. The variable d_{24}^- represents negative deviation from the goal, while d_{23}^- is the amount of overachievement of this goal. The constraint is given as:

$$X_{1,1} + d_{24}^- - d_{24}^+ = 100$$

(iv) In order to avoid gross inequalities of demand satisfaction among the various depots, the goal of satisfying at least 80% of each depot's demand is included. The goal constraints are given as follows:

$$\sum_{i=1}^{3} X_{i,1} + d_{25}^- - d_{25}^+ = 160$$

$$\sum_{i=1}^{3} X_{i,2} + d_{26}^- - d_{26}^+ = 80$$

$$\sum_{i=1}^{3} X_{i,3} + d_{27}^- - d_{26}^+ = 80$$

$$\sum_{i=1}^{3} X_{i,4} + d_{28}^- - d_{28}^+ = 80$$

$$\sum_{i=1}^{3} X_{i,5} + d_{29}^- - d_{29}^+ = 80$$

$$\sum_{i=1}^{3} X_{i,6} + d_{30}^- - d_{30}^+ = 160$$

$$\sum_{i=1}^{3} X_{i,7} + d_{31}^- - d_{31}^+ = 240$$

$$\sum_{i=1}^{3} X_{i,8} + d_{32}^- - d_{32}^+ = 160$$

$$\sum_{i=1}^{3} X_{i,9} + d_{33}^- - d_{33}^+ = 80$$

$$\sum_{i=1}^{3} X_{i,10} + d_{34}^- - d_{34}^+ = 160$$

$$\sum_{i=1}^{3} X_{i,11} + d_{35}^- - d_{35}^+ = 80$$

$$\sum_{i=1}^{3} X_{i,12} + d_{36}^- - d_{36}^+ = 160$$

$$\sum_{i=1}^{3} X_{i,13} + d_{37}^- - d_{37}^+ = 160$$

$$\sum_{i=1}^{3} X_{i,14} + d_{38}^- - d_{38}^+ = 80$$

$$\sum_{i=1}^{3} X_{i,15} + d_{39}^- - d_{39}^+ = 200$$

$$\sum_{i=1}^{3} X_{i,16} + d_{40}^- - d_{40}^+ = 120$$

$$\sum_{i=1}^{3} X_{i,17} + d_{41}^- - d_{41}^+ = 120$$

$$\sum_{i=1}^{3} X_{i,18} + d_{42}^- - d_{42}^+ = 400$$

$$\sum_{i=1}^{3} X_{i,19} + d_{43}^- - d_{43}^+ = 560$$

$$\sum_{i=1}^{3} X_{i,20} + d_{44}^- - d_{44}^+ = 560$$

(v) Minimise the total transportation cost not greater than the budgeted, Rs. 590, 415.

$$\sum_{i=1}^{3}\sum_{j=1}^{20} C_{ij} X_{ij} + d_{45}^- - d_{45}^+ = 590,415, \text{ all } i, j$$

(vi) The company's goal is to minimise shipment of goods from plant 3 to depot 5, given the road conditions along that route. Thus, the goal for the constraint is set to zero, with d_{46}^+ minimised.

$$X_{3,5} - d_{46}^+ = 0$$

(vii) It is desired to transport required amounts to depots 2 and 7 such that an equal portion of demand for each is satisfied. This can be expressed as:

$$(X_{1,2} + X_{2,2} + X_{3,2}) / 100 = (X_{1,7} + X_{2,7} + X_{3,7}) / 300$$

Thus, the goal constraint becomes:

$$X_{1,2} + X_{2,2} + X_{3,2} - 0.333\,(X_{1,7} + X_{2,7} + X_{3,7}) + d_{47}^- - d_{47}^+ = 0$$

(viii) If C_{ij} is denoted as the unit transportation cost from the ith plant to the jth depot the total transportation is given by $C_{ij} X_{ij}$, for all i and j. Since the Company wishes to minimise total transportation costs, a goal of zero cost is set and an attempt is made to minimise the positive deviation from this goal specified.

$$\sum_{i=1}^{3}\sum_{j=1}^{20} C_{ij} X_{ij} - d_{48}^+ = 0, \text{ for all } i, j$$

THE OBJECTIVE FUNCTION

The management ranked its goals based on the range of importance as P_1 to P_7. The complete LGP model for the problem is formulated as follows:

$$\begin{aligned}\text{Minimise } Z = {} & P_1 d_{21}^- + P_2 d_{24}^- + P_3(d_{25}^- + d_{26}^- + d_{27}^- \\ & + d_{28}^- + d_{29}^- + d_{30}^- + d_{31}^- + d_{32}^- \\ & + d_{33}^- + d_{34}^- + d_{35}^- + d_{36}^- + d_{37}^- \\ & + d_{38}^- + d_{39}^- + d_{40}^- + d_{41}^- + d_{42}^- \\ & + d_{43}^- + d_{44}^-) + P_4 d_{45}^+ + P_5 d_{46}^+ \\ & + P_6(d_{47}^- + d_{47}^+) + P_7 d_{48}^+\end{aligned}$$

subject to equations 1 – 48, X_{ij}, d_i^-, $d_i^+ \geq 0$

Results

The present GP transportation problem contains 60 variables, 48 constraints, 7 priorities, and an objective function. The solution of the problem is obtained by using the *QSB* + software package. The results are as follows:

Real Variables	Deviational Variables
$X_{1,1} = 100$	$d_3^- = 500$
$X_{1,2} = 70$	$d_4^- = 40$
$X_{1,3} = 80$	$d_5^- = 20$
$X_{1,4} = 80$	$d_6^- = 20$
$X_{1,5} = 160$	$d_7^- = 20$
$X_{1,7} = 10$	$d_8^- = 40$
$X_{1,8} = 160$	$d_9^- = 40$
$X_{1,13} = 60$	$d_{11}^- = 40$
$X_{1,14} = 80$	$d_{12}^- = 20$
$X_{1,18} = 500$	$d_{13}^- = 40$
$X_{2,6} = 160$	$d_{14}^- = 20$
$X_{2,9} = 80$	$d_{15}^- = 40$
$X_{2,10} = 160$	$d_{16}^- = 40$
$X_{2,11} = 80$	$d_{17}^- = 20$
$X_{2,12} = 160$	$d_{18}^- = 50$
$X_{2,13} = 100$	$d_{19}^- = 30$
$X_{2,15} = 200$	$d_{20}^- = 30$
$X_{2,20} = 560$	$d_{22}^- = 140$
$X_{3,1} = 60$	$d_{23}^- = 140$
$X_{3,2} = 10$	$d_{42}^+ = 100$
$X_{3,8} = 230$	$d_{45}^- = 72475$
$X_{3,16} = 120$	$d_{48}^+ = 517940$
$X_{3,17} = 120$	
$X_{3,19} = 560$	

All other real and deviational variables are zero.

The results obtained reveal that the optimal solution of all objectives were achieved.

CONCLUSION

In this study, we have been able to demonstrate in line with previous studies [3, 5] that the GP approach is improved technique over single objective criterion when multiple conflicting objectives are involved. We acknowledge that the minimisation of total transportation cost in munufacturing establishments is vital but not a sufficient condition to gurarantee optimal operational performance when other attendant factors like transportation sehedule, union contracts, stable employment condition, transportation hazards, etc. that play important roles in transportation problems are present.

We obtained optimal solution for all seven objective junctions. This marginal result on total transport cost minimisation points to the imperativeness of some trade-off with a given desired policy. Significantly, the study reinforces the notion that managers chould critically review the priority structure for the goal in their establishment. Besides, the use of the goal programming approach in this study provides an excellent opportunity for the manager to include non-quantifiable information perferences into the decision.

REFERENCES

1. Fong *C O* and Rao *M R* (1975) *parametric Studies in Transportation Type problems.* Naval Res. Logist. Q., 22, 355 – 364.
2. Kwak *N K* and Schniederjans *M J* (1985) *Goal programming solutions to Transportation Problems with Variable Supply and Demand Requirements.* Socio-Econ. Planning Science, 19(2), 95 – 100.
3. Kwak *N K* and Schniederjans *M J* (1979) *A Goal Programming Model for Improved Transporation Problem Solutions.* Omegea, 12, 367 – 370.
4. Lee *S M* (1972) *Goal Proggramming for Decision Analysis. Auerbach, Philadelphia, U.S.A.*
5. Lee *S M* and Moore *L J* (1973) *Optimization of Transportation Problems with Multiple Objectives.* AIIE Trans. 5(4), 333 338.
6. Pal *B B* and Basu *M* (1984) Optimal Slution of a Transportation Problems by Goal Programming Technique. Bull. Cal. math. Soc., 76, 342 – 348.
7. Srinivasan V and Thompson *G L* (1973) *Determining Optimal Growth Paths in Logistics Operation.* Naval. Res. Logist. Q., 20, 265 – 269.